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STANDARD – XI

MATHS

[Sets, Binomial Theorm, Inequality, Complex Number, Sequence – Series, Trigonometry]

TIME : 1.0 Hr MARKS : 50

NAME : _____ Marks Obtained : ____

Q – 1 Solve the following

1	**In a class of 32 students, each of whom must take English or Hindi; 20 take English, 9 take both English and Hindi. How many take Hindi?**
2	**In a class of 50 students, 22 like History, 25 like Geography and 10 like both subjects. Draw a Venn diagram and find the number of students who** (*i*) do not like History (*ii*) do not like Geography (*iii*) like neither History nor Geography
3	If the first three terms in the expansion of $(1 + ax)^n$ in ascending powers of x are $1 + 12x + 64x^2$, find n and a. **[SC]**
4	**Write the middle term or terms in the expansion of** $\left(3a - \frac{a^3}{6}\right)^9$
5	**Find the term independent of x in the expansion of** $\left(\frac{3}{2}x^2 - \frac{1}{3x}\right)^9$
6	**How many terms of a G.P. 3, 3^2, 3^3, ... are needed to give the sum 120 ?**
7	**Express (*i*) 1 radian, (*ii*) $\frac{\pi}{3}$ radians, (*iii*) $\frac{\pi}{15}$ radians in degrees.** **Express (*i*) 45°, (*ii*) 30°, (*iii*) 9° in radians.**
8	Prove that $\cos^2\frac{\pi}{8} + \cos^2\frac{3\pi}{8} + \cos^2\frac{5\pi}{8} + \cos^2\frac{7\pi}{8} = 2.$
9	Find the general value of theta if **(*i*) $2\sin\theta - 1 = 0$** **(*ii*) $\sin\theta = -\frac{\sqrt{3}}{2}$** **(*iii*) $\cos\theta = \frac{1}{\sqrt{2}}$** **(*iv*) $\cos\theta = -\frac{1}{2}$**
10	Find all the values of θ satisfying the equation $\cos 2\theta - \cos 8\theta + \cos 6\theta = 1$, such that $0 \le \theta \le \pi$.
11	Find the complex number z satisfying the equation $\left\|\frac{z-12}{z-8i}\right| = \frac{5}{3}$, $\left\|\frac{z-4}{z-8}\right| = 1$.

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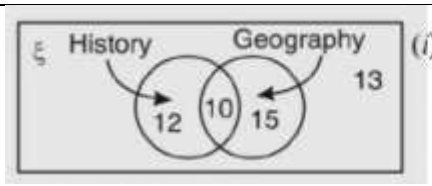
12	Find the modulus and amplitude of $\frac{(1+i)(2+i)}{(3+i)}$
13	Change the complex numbers into polar form $\frac{1+2i}{1-(1-i)^2}$
14	**If 1, ω, ω² are cube roots of unity, prove that** $(x+y)^2 + (x\omega + y\omega^2)^2 + (x\omega^2 + y\omega)^2 = 6xy.$
15	**The sum of three consecutive numbers of a G.P. is 56. If we subtract 1, 7 and 21 from these numbers in the order, the resulting numbers form an A.P, find the numbers.**
16	**Find the value of $0.\overline{423}$.**
17	**The letters of the word 'RANDOM' are written in all possible ways and these words are written out as in a dictionary. Find the rank of the word 'RANDOM'.**
18	How many different words can be formed of the letters of the word '*MALENKOV*' so that (*i*) no two vowels are together, (*ii*) the vowels may occupy odd places?
19	How many committees of 5 members each can be formed with 8 officials and 4 non-official members in the following cases: (*i*) each consists of 3 officials and 2 non-official members; (*ii*) each contains at least two non-official members; (*iii*) a particular official member is never included; (*iv*) a particular non-official member is always included?
20	The second term of a G.P. is 2 and the sum of infinite terms is 8. Find the first term.
21	$\frac{\sin 150^\circ - 5\cos 300^\circ + 7\tan 225^\circ}{\tan 135^\circ + 3\sin 210^\circ}$.
22	Find the value of $\sin(\alpha+\beta)$, $\cos(\alpha+\beta)$, and $\tan(\alpha+\beta)$, given (*i*) $\sin\alpha = \frac{3}{5}$, $\cos\beta = \frac{5}{13}$, α and β in Quadrant I.
23	Prove that $\cos 20^\circ \cos 40^\circ \cos 80^\circ = \frac{1}{8}$
24	Prove that $2\cos\frac{\pi}{13}\cos\frac{9\pi}{13} + \cos\frac{3\pi}{13} + \cos\frac{5\pi}{13} = 0.$
25	**Prove that:** $\sqrt{2+\sqrt{2+\sqrt{2+2\cos 8\theta}}} = 2\cos\theta$, **where** $\theta \in \left[\frac{-\pi}{8}, \frac{\pi}{8}\right]$

ANSWERS

1	21
2	 (*i*) 28 (*ii*) 25 (*iii*) 13
3	$9, \frac{4}{3}$
4	$-\frac{21}{16}a^{19}$
5	$\frac{7}{18}$
6	**Sol.** Here, $a = 3, r = 3, S_n = 120$ Now, $S_n = \frac{a(r^n - 1)}{r - 1}, r > 1$ $\Rightarrow 120 = \frac{3(3^n - 1)}{3 - 1}$ $\Rightarrow 120 = \frac{3(3^n - 1)}{2}$ $\Rightarrow 120 \times 2 = 3(3^n - 1)$ $\Rightarrow 3^n - 1 = 80 \Rightarrow 3^n = 80 + 1$ $\Rightarrow 3^n = 81 \Rightarrow 3^n = 34$ On comparing the power of 3 both sides, $\Rightarrow n = 4$
7	$1 \text{ rad} = 57°17'45'', \frac{\pi}{3} \text{ rad} = 60°; \frac{\pi}{15} \text{ rad} = 12°$ $45° = \frac{\pi}{4}\text{rad}; \quad 30° = \frac{\pi}{6}\text{rad}; \quad 9° = \frac{\pi}{20}\text{rad}.$
8	Excercie – 5(d) Ex 24

9	$(i)\ 2\sin\theta - 1 = 0 \Rightarrow \sin\theta = \frac{1}{2} = \sin\frac{\pi}{6}$. Hence, $\theta = n\pi + (-1)^n \frac{\pi}{6}, n \in I$ $(ii)\ \sin\theta = -\frac{\sqrt{3}}{2} = \sin(-\frac{\pi}{3})$. Hence, $\theta = n\pi + (-1)^n(-\frac{\pi}{3}) = n\pi - (-1)^n\frac{\pi}{3} \in I$ $iii)\ \cos\theta = \frac{1}{\sqrt{2}} = \cos\frac{\pi}{4}$. Hence, $\theta = 2n\pi \pm \frac{\pi}{4}, n \in I$. $(iv)\ \cos\theta = -\frac{1}{2} = \cos\left(\pi - \frac{\pi}{3}\right) = \cos\frac{2\pi}{3}$. Hence, $\theta = 2n\pi \pm \frac{2\pi}{3}, n \in I$.
10	Exercise – 6 / Ex 26 / Ans $0, \frac{\pi}{8}, \frac{3\pi}{8}, \frac{5\pi}{8}, \frac{7\pi}{8}, \frac{\pi}{3}, \frac{2\pi}{3}, \pi$
11	Exercise – 9 C / Ex 7 / Ans $6 + 8i, 6 + 17i$
12	Modulus = 1, argument = 53°8′
13	$\cos 0° + i \sin 0°$
14	Complex no. Example sum 64 (iv)
15	**Sol.** Let three consecutive numbers of a GP be a, ar, ar^2. Then according to the given condition.

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$a + ar + ar = 56$ ...(1)

Also, it is given if we subtract 1, 7 and 21 from consecutive number, it forms a AP

$$\therefore (ar-7) = \frac{(a-1)(ar^2-21)}{2}$$

$\Rightarrow 2(ar-7) = a + ar^2 - 22$

$\Rightarrow ar^2 - 2ar + a = +8$ ...(ii)

Divide Eq. (ii) by Eq. (i) we get

$$\frac{a(r^2-2r+1)}{a(1+r+r^2)} = \frac{8}{56}$$

$7(r^2 - 2r + 1) = (1 + r + r^2)$

$\Rightarrow 6r^2 - 15r + 6 = 0$

$\Rightarrow 3(2r^2 - 5r + 2) = 0$

$\Rightarrow 3[(2r-1)(r-2)] = 0$

$$\Rightarrow r = \frac{1}{2}, 2$$

When $r = \frac{1}{2}$

$$a + a\left(\frac{1}{2}\right) + a\left(\frac{1}{4}\right) = 56$$

$$\frac{7a}{4} = 56 \Rightarrow a = 32$$

When $r = 2$, $a + 2a + 4a = 56$

$7a = 56 \Rightarrow a = 8$

When $a = 32$, $r = \frac{1}{2}$, then numbers are

$$32,\ 32 \times \frac{1}{2},\ 32 \times \frac{1}{4} \quad i.e.,\ 32, 16, 8$$

When $a = 8$, $r = 2$, then numbers are $8, 8 \times 2, 8 \times 4$ *i.e.*, 8, 16, 32

Hence, the required numbers will be 8, 16, 32

16 **Sol.** $0.4\overline{23} = 0.423\ 23\ 23 \ldots$ ad inf.

$= 0.4 + 0.023 + 0.00023 + \ldots$ ad inf.

$$= \frac{4}{10} + \frac{23}{1000} + \frac{23}{100000} + \ldots \text{ad inf.}$$

$$= \frac{4}{10} + \frac{23}{10^3}\left(1 + \frac{1}{10^2} + \frac{1}{10^4} + \ldots \text{ad inf.}\right)$$

$$= \frac{4}{10} + \frac{23}{10^3} \times \frac{1}{1-\left(\frac{1}{10^2}\right)} = \frac{4}{10} + \frac{23}{1000} \times \frac{100}{99} = \frac{419}{990}.$$

17 Example – 29 / Ans = 614

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18	$(i)\ 14400\ \ (ii)\ 2880$
19	$(i)\ 336\ \ (ii)\ 456\ \ (iii)\ 462\ \ (iv)\ 330$
20	4
21	(-2)
22	$(i)\ \frac{63}{65}, \frac{-16}{65}, \frac{-63}{16}$
23	
24	
25	$\text{L.H.S.} = \sqrt{2+\sqrt{2+\sqrt{2+2\cos 8\theta}}} = \sqrt{2+\sqrt{2+\sqrt{2\,(2\cos^2 4\theta)}}}$ $= \sqrt{2+\sqrt{2+2\cos 4\theta}} = \sqrt{2+\sqrt{2(1+\cos 4\theta)}}$ $= \sqrt{2+\sqrt{2(2\cos^2 2\theta)}} = \sqrt{2+2\cos 2\theta}$ $= \sqrt{2(1+\cos 2\theta)} = \sqrt{2\cdot(2\cos^2\theta)} = \sqrt{4\cos^2\theta} = 2\,\lvert\cos\theta\rvert$ $= 2\cos\theta \text{ as } \theta \in \left[\frac{-\pi}{8}, \frac{\pi}{8}\right] \Rightarrow \cos\theta > 0 \Rightarrow \lvert\cos\theta\rvert = \cos\theta.$

Q – 1 Solve the following [14]

1 Evaluate: $\sum_{n=1}^{13} (i^n + i^{n+1})$, where $n \in \mathbb{N}$.

2 Let $A = \{x, y, z\}$ and $B = \{1, 2\}$. Find the number of relations from A to B.

3 In how many ways can the letters of the word STRANGE be arranged to form the words beginning with T?

4 If α and β are the roots of the quadratic equation $px^2 + qx + r = 0$, then form the quadratic equa whose roots are

(i) $\frac{1}{\alpha}, \frac{1}{\beta}$. (ii) $\alpha^2\beta, \alpha\beta^2$.

5 Find the domain of the following functions:

(i) $f(x) = \frac{1}{\sqrt{x - |x|}}$. (ii) $f(x) = \frac{1}{\sqrt{x + |x|}}$.

6 Solve for real x: $|3 - 4x| \geq 9$.

7 A survey of 500 television viewers produced the given information: 285 watch football, 195 w hockey, 115 watch cricket, 45 watch football and cricket, 70 watch football and hockey, 50 watch cri and hockey, 50 do not watch any of the three games. How many watch exactly one of the three gam

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Q – 2 Find the modulus and amplitude of following and express in polar form [6]

1 In how many ways 3 mathematics books, 4 history books, 3 chemistry books and 2 biology b can be arranged on a shelf so that all books of the same subjects are together.

2 $\frac{(3+4i)(4+5i)}{(4+3i)(6+7i)}$

Q – 3 Prove the following [6]

1 $(1-\omega+\omega^2)(1+\omega-\omega^2)(1-\omega-\omega^2) = 8.$

2 $\frac{a+b\omega+c\omega^2}{c+a\omega+b\omega^2}+\frac{a+b\omega+c\omega^2}{b+c\omega+a\omega^2}=-1$

3 Find the square root of complex number z = 3 + 4i

Q – 4 Prove the following [14]

1 Express the following angles in degrees : $\frac{\pi}{6}, \frac{14}{15}\pi, \frac{11}{18}\pi, \frac{7}{90}\pi.$

2 Express the following angles in radians (*i*) 1′ (*ii*) 20° (*iii*) 135°.

3 The minute hand of a clock is 15 cm long. How far does the tip of the hand move during 40 minutes ? (Take $\pi = 3.14$)

4 A horse is tethered to a stake by a rope 810 cm long. If the horse moves along the circumfere of a circle always keeping the rope tight, find how far it will have gone when the rope has tra out an angle of 70°?

5 **2000 candidates appear in a written test in Mathematics and Gerneral Awareness for a Government job. 1800 passed in at least one subject. If 1200 passed in Mathematics and 1500 in General Awareness find:**

(*i*) how many passed in both the subjects?

(*ii*) how many passed in Mathematics only?

(*iii*) how many failed in General Awareness?

6 In a class of 150 students, the following results were obtained in a certain examination; 45 students failed in Maths, 50 students failed in Physics, 48 students failed in Chemistry, 30 students failed in both Maths and Physics, 32 failed in Physics and Chemistry. 35 failed in both Maths and Chemistry, 25 failed in all the three subjects. Draw a Venn diagram corresponding to this data and find the number of students who have failed in at least one subject.

ANSWERS

Q – 1 Solve the following

(1) Ans 8,

(2) **7.** $q\,(p^2 - 2q),\ p^4 + 3q^2 - 4p^2q.$

(3) **10.** (*i*) $\dfrac{b^2\,(b^2 - 4ac)}{a^2c^2}$ (*ii*) $\dfrac{b^4 + 2a^2c^2 - 4ab^2c}{a^3c}.$

(4) (*i*) $f = \{(2, 5), (3, 7), (4, 6)\}$, (*ii*) $g = \{(2, 2), (3, 5), (4, 2)\}$
(*iii*) $h = \{(2, 2), (5, 3), (6, 4), (7, 4)\}.$

(5) **4.** –5, –2

(6) K = 2

(7) Domain = $R - \{2\}$, Range is $R - \{4\}$

Q – 2 Find the modulus and amplitude of following and express in polar form

9. Modulus = $\sqrt{170}$, argument = 32°29′

Modulus = $\sqrt{\dfrac{41}{85}}$, argument = 18°12′

Q – 3

(Ans ±(2 + i)]

Q – 4

(1) 30°, 168°, 110°,14°.

(2) $\dfrac{\pi}{10800}, \dfrac{\pi}{9}, \dfrac{3\pi}{4}.$

(3) 62.8 cm

(4) 990 m

(5) (*i*) 900 (*ii*) 300 (*iii*) 500

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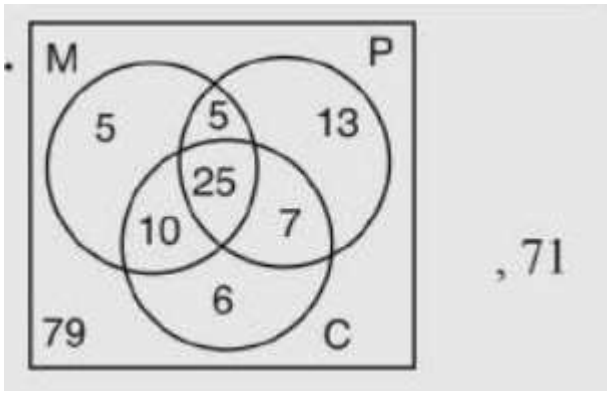
(6) 

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